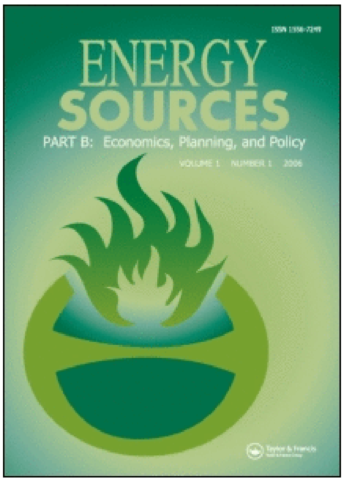

Energy Sources, Part B: Economics, Planning, and Policy

Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t716100766

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Online publication date: 30 March 2010

To cite this Article Oguejiofor, G. C.(2010) 'Modeling of Linear and Exponential Growth and Decay Equations and Testing Them on Pre- and Post-War-Coal Production in Nigeria: An Operations Research Approach', Energy Sources, Part B: Economics, Planning, and Policy, 5: 2, 116 — 125

To link to this Article: DOI: 10.1080/15567240802053459
URL: http://dx.doi.org/10.1080/15567240802053459

Energy Sources, Part B, 5:116–125, 2010

ISSN: 1556-7249 print/1556-7257 online
DOI: 10.1080/15567240802053459

Modeling of Linear and Exponential Growth and Decay Equations and Testing Them on Pre- and Post-War-Coal Production in Nigeria: An Operations Research Approach

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Abstract *Despite Nigeria's huge reserves in coal resource, the vanishing coal production over the years has been a source of grave concern to many who are interested in witnessing the diversification of Nigeria's energy sector to non-petroleum resources. Within the framework of Operations Research, linear growth and decay equations on one hand, and exponential growth and decay equations on the other hand, were modeled for coal production in Nigeria. Subsequently, the growth and decay models for linear and exponential considerations were tested on the annual coal production data with a view to establishing the appropriate characters/features of the profiles of Nigeria coal production from 1916 to 2001, under two scenarios, namely: Scenario 1—pre-Nigerian civil war period, 1916–1966; and Scenario 2—post-Nigerian civil war period, 1973–2001. The findings gave fascinating insights into the features of the Nigerian coal production. Thus in a nutshell, it was found that the pre-war period of coal production, 1916–1966 approximated to the linear growth trend. While the post-war period of coal production, 1973–2001 was close to the feature of exponential-decay curve. It is expected that the findings obtained from this investigation would help in planning for the rebirth of Nigerian coal production and its utilization in energy generation.*

Keywords exponential-decay curve, linear-growth trend, mathematical-growth-and-decay models, non-petroleum resources, scenarios, vanishing-coal production

1. Introduction

The National Integrated Power Programme (NIPP) was conceptualized in 2004 by the Federal Government of Nigeria. It is a fast-track approach to realizing the national objectives of improving the nation's power supply capacity by expanding its electricity infrastructure through a number of integrated power generation, transmission, and distribution, as well as gas supply projects. Udo (2007) writes that power projects initiated under the NIPP were categorized into two groups:

- the gas-fired thermal station, and
- the hydro-electric power plants.

The coal-fired thermal station, which formed the bedrock of Nigeria's early industrialization and economic development from 1916 up to 1959 when commercial

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petroleum was discovered, was not in the map of the NIPP project categorization. Thus, coal that is supposed to complement petroleum resource as an alternative energy source continued to witness vanishing output over the years and decades. On the other hand, the petroleum resource, which was given prominence in the NIPP project categorization, suffers relentless disruption of supplies to gas-fired-power stations from Niger-Delta insurgencies/militancies. All these have impacted heavily on the protracted energy crisis in Nigeria.

Nigeria's high-grade coal reserve put at 5 billion tonnes by RMRDC (1997) is so promising as to attract a rebirth of the Nigerian coal industry for the purpose of enhancing power generation and complementing petroleum-fired power stations. However, the rebirth of the coal industry in Nigeria may require a hindsight or retrospective investigation of the past coal production levels and trends in Nigeria. The expectation of the hindsight investigation is to provide useful hints and tips required for articulating a viable plan of action for the rebirth of coal production in the short-run and the sustenance of coal production in the long-term arrangement.

To explore the benefit of hindsight, this investigation will delve into applying linear-mathematical and exponential-mathematical models as tools of operations research (OR) for investigating coal production data from 1916 to 2001. The outbreak of the Nigerian civil war disrupted coal production from 1967 to 1970, giving rise to two coal production scenarios, namely:

- Scenario 1: Pre-war coal production, 1916–1966
- Scenario 2: Post-war coal production, 1973–2001

Therefore, to accomplish this investigation under the above-named scenarios, this article is broken up into the following objects:

a. Literature review on linear and exponential growth/decay models.
b. Method and tools of evaluation of coal production data.
c. Presentation of the results obtained.
d. Discussion of the results and findings.
e. Conclusion.

2. Review of Relevant Literature

This literature review would be undertaken from the following categorization:

- linear growth and decay models, and
- exponential growth and decay models.

2.1. Linear Growth and Decay Models

For a first-degree relationship, Stroud (2001) writes that the equation of a straight line can always be expressed in the form:

$$Y = mx + c \tag{1}$$

where

$m =$ the gradient $= \frac{dy}{dx}$,
$c =$ the intercept.

Applying Eq. (1) to coal production in Scenario 1, and letting c = coal output in 1916, $Y = P$ = coal output in subsequent years up to 1966, and $x = t$ = time in years, Eq. (1) can be rewritten for growth in coal production as:

$$P = mt + c \tag{2}$$

where $m = \frac{dP}{dt}$.

This linear-growth function will be applied in the investigation of Scenario 1, that is the pre-war coal production period covering 1916 to 1966.

Similarly, Eq. (1) can be rewritten for decline (decay) in coal production as:

$$P = -mt + c \tag{3}$$

where the gradient (m) is negative.

Equation (3), which is a linear-decay function, will be used for the evaluation of Scenario 2 involving the post-war coal production period 1973 to 2001.

2.2. *Exponential Growth and Decay Models*

Barrett et al. (1991) states that exponential decay can be described by the equation:

$$P_t = P_o e^{kt} \tag{4}$$

where

P_t = the number of radioactive atoms present at time, t,
P_o = the number of radioactive atoms present initially when time, $t = 0$, and
k = negative constant.

This applies in radioactive decay of radioactive substances.

Using the coal production as a model, and letting P_o = coal output in 1916, P_t = coal output in subsequent years up to 1966, k = growth constant, t = time in years, then the exponential growth model can be expressed as:

$$P_t = P_o e^{kt} \tag{5}$$

For models in the form of Eq. (5), Porteous (1991) describes the growth constant (k) as exponential-growth rate and the plot arising from it as exponential-growth curve. Also, Miller (1998) writes that exponential growth typically yields a curve shaped like the letter J. Equation (5) will be applied in investigating Scenario 1 for coal production, which spans over 1916–1966.

By the same token, Eq. (4) can be modeled for the post-war coal production (Scenario 2). By letting P_o = coal output in 1973, P_t = coal output in subsequent years up to 2001, k = decay coefficient, t = time in years, the exponential decay model becomes:

$$P_t = P_o e^{-kt} \tag{6}$$

Porteous (1991) depicts the equation of the above form as exponential-decay model and the graph from it as exponential decay curve. Stroud (2001) shows that the graph of exponential growth is the reciprocal of exponential decay.

3. Methods and Tools

3.1. Data and Their Sources

The study specimens are the annual coal production data starting from 1916, when coal production started in the Enugu Coal Mines, to 2001, when skeletal production was merely maintained. These specimens are displayed in the Appendix of this report and were compiled from the literature shown at the bottom of the Appendix.

3.2. Computational Tools and Methods

The study specimens in the Appendix were subjected to mathematical evaluations by the application of the study tools, namely Eqs. (2), (3), (5), and (6), respectively. The annual values obtained from the computations are categorized as follows:

- Scenario 1 covering 1916 to 1966: Linear-growth and exponential-growth values obtained from computations using Eqs. (2) and (5), respectively.
- Scenario 2 spanning from 1973 to 2001: Linear-decay and exponential-decay values obtained by the exploitation of Eqs. (3) and (6) in the calculation of the values.

3.3. Method of Analysis

From the values computed, graphs whose values were derived from linear and exponential models were superimposed on the plots of actual coal production. Examples, under scenario 1, linear-growth and exponential-growth curves were imposed on the plot of actual coal production for the period 1916–1966. Also, under Scenario 2, linear-decay and exponential-decay graphs were imposed on the curve of actual coal production for 1973–2001 periods.

Comparisons were made among the three plots for each scenario, in order to ascertain which of the superimposed curves was consistent or resembled or approximate the trends of the actual coal production. In a nutshell, this was how the graphs were used as analytical tools for this investigation as the Discussion section would show.

4. Results Obtained

Table 1 displays the values obtained from the linear-growth and exponential-growth computations. See Eqs. (2) and (5). Also Figure 1 shows the trends of linear-growth values, exponential-growth values, and actual coal production over time (1916–1966).

Similarly, Table 2 presents the values realized from the calculations from linear-decay and exponential-decay equations. Refer to Eqs. (3) and (6). While Figure 2 portrays the profiles of linear-decay values, exponential-decay values and actual coal production from the peak production of 323,001 metric tonnes recorded in 1973 to 2,712 metric tonnes recorded in 2001 production.

5. Discussion

5.1. Discussion of Graphs

A comparison among the trends in Figure 1 and Table 1 could be approached from two perspectives, namely: (a) a comparison between the actual coal production profile and

Table 1
Values (linear and exponential) from computations for growing coal production 1916–1966 (Scenario 1)

Year	Time, *t*, years	Production, metric tonnes	Values from linear equation, metric tonnes	Values from exponential equation, metric tonnes
1916	0	24,511	24,511	24,511
1918	2	145,407	52,594	28,085
1920	4	180,122	80,677	32,177
1922	6	112,818	108,760	36,864
1924	8	220,161	136,843	42,235
1926	10	353,274	164,926	48,388
1928	12	363,743	193,009	55,437
1930	14	327,681	221,092	63,513
1932	16	259,860	249,175	72,766
1934	18	258,893	277,258	83,366
1936	20	310,308	305,341	95,511
1938	22	323,266	333,424	109,426
1940	24	318,594	361,507	125,367
1942	26	463,978	389,590	143,631
1944	28	668,158	417,672	164,555
1946	30	633,852	445,755	188,528
1948	32	610,283	473,838	215,993
1950	34	583,487	501,921	247,459
1952	36	613,374	530,004	283,509
1954	38	675,918	558,087	324,811
1956	40	790,030	586,170	372,131
1958	42	905,397	614,253	426,343
1960	44	565,681	642,336	488,454
1962	46	615,681	670,419	559,613
1964	48	698,502	698,502	641,138
1966	50	730,183	726,585	734,540

the exponential-growth trends and (b) a comparison between the actual coal profile and the linear-growth trend.

The actual coal production profile does not respond to the exponential-growth trend as there exists a significant deviation in both the magnitude of the data (see Table 1) and the features of the plots (see Figure 1). This is a pointer that the actual and exponential trends are inconsistent. Therefore, it may not be appropriate to describe the coal production profile for Scenario 1 as an exponential-growth curve.

However, the linear-growth values appear like an averaging of the actual coal production data (see Table 1). Thus the linear-growth trend skews towards the actual coal production profile (refer to Figure 1). Therefore, it could be rightly said that the linear-growth trend approximates to the actual coal production profile.

Similarly, an examination of various trends portrayed in Figure 2 and Table 2 suggests that the exponential-decay curve fairly averages with the actual production plot; while the linear-decay plot is at variance with the actual production pattern. Therefore, it

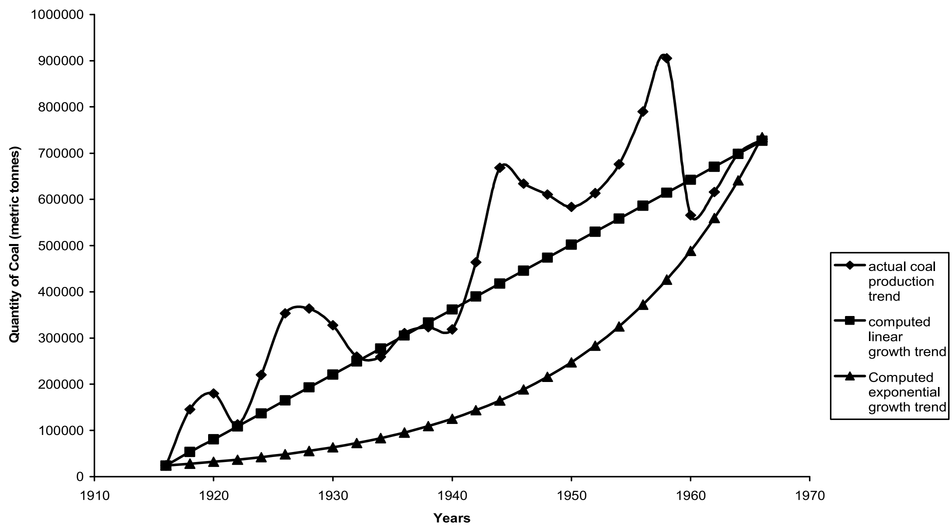

Figure 1. Graphs of the growing trends of coal production from 1916 to 1966.

may be proper describing or qualifying the coal production profile for Scenario 2 as an exponential–decay curve, as Figure 2 and Table 2 suggest.

5.2. Discussion of Findings

The finding from Figure 1 is that coal production from 1916 to 1966 witnessed linear-growth trend. Interestingly, linear-growth profile from 1916–1966 depicts the increasing relevance and demand for coal as the main engine of Nigeria's industrialization and economic development. For example, the Nigeria Railway Corporation (NRC), the Electric Corporation of Nigeria (ECN), Nigerian Cement Company (NIGERCEM), and the Marine Department depended on coal for firing steam locomotive engines, thermal power plants, clement kilns, and ships/steamers. It was, therefore, not surprising that the relevance and demand for coal maintained an average steady upsurge in production that is consistent with the linear-growth plot (Figure 1) evident from 1916 to 1966.

The second finding established from Figure 2 is that coal production from 1973 to 2001 experienced an exponential-decay curve. This portrays the diminishing relevance and demand for coal as the driver for Nigeria's industrialization and economic development. Petroleum, which was discovered in 1957 and produced in commercial quantity in the 1970s, overtook and overwhelmed coal resources. The result was the neglect of coal production in preference to petroleum resource. So coal production witnessed dwindling output from 1973 to 2001 that responded to the exponential-decay curve.

5.3. Furthering the Investigation

The quote of wisdom from Tom Stoppard says that "every exit is an entry somewhere else." Therefore, further investigation could involve the application of Correlation Analysis. The two most frequently used simple correlation coefficients are:

(a) Pearson Product Moments Correlation Coefficient
(b) Spearman Rank Order Correlation Coefficient

Table 2
Values (linear and exponential) from computations for declining coal production 1973–2001 (Scenario 2)

Year	Time, *t*, years	Production, metric tonnes	Values from linear equation, metric tonnes	Values from exponential equation, metric tonnes
1971	—	24,400	—	—
1973	0	323,001	323,001	323,001
1975	2	250,769	300,123	229,443
1977	4	249,446	277,245	162,918
1979	6	188,806	254,367	115,728
1981	8	114,875	231,489	82,207
1983	10	52,730	208,611	58,396
1985	12	139,744	185,733	41,481
1987	14	110,161	162,855	29,466
1989	16	80,973	139,977	20,931
1991	18	51,238	117,099	14,869
1993	20	27,686	94,221	10,562
1995	22	19,505	71,343	7,503
1997	24	20,766	48,465	5,329
1999	26	12,136	25,587	3,786
2001	28	2,712	2,709	2,689

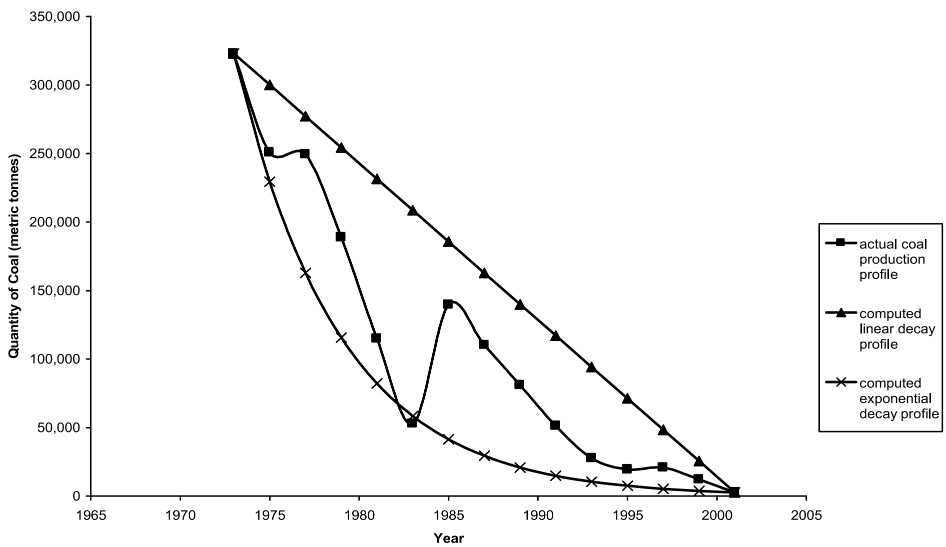

Figure 2. Plots of the decaying profiles of coal production from 1973 to 2001.

Pearson Product Moments Correlation Coefficients may be employed in studying the correlation between the actual coal production data and linear/exponential values for growth and decay considerations. In this regard, two approaches may be available. In deviation from mean approach, Pearson Product Moments Correlation Coefficient gives that:

$$r = \frac{\sum(x-\overline{x})(y-\overline{y})}{\sqrt{\sum(x-\overline{x})^2\sum(y-\overline{y})^2}}$$

From the raw score approach, Pearson Product Moments Correlation Coefficient gives that:

$$r = \frac{N\sum xy - \sum x \sum y}{\sqrt{\left[N\sum x^2 - (\sum x)^2\right]\left[N\sum Y^2 - (\sum Y)^2\right]}}$$

where x = actual coal production data and y = linear or exponential values.

6. Conclusion

Energy is crucial to economic development, quality education, and industrialization. This investigation has successfully shed light on the nature of the character and feature of Nigerian coal production. Indeed, this study has established the appropriate word/phrasing for describing/qualifying the patterns/trends of Nigerian coal production under the scenarios of growth (1916–1966) and decline (1973–2001). Also, the investigation has established that the growth period of coal production of Nigeria (1916–1966) approximates to the linear growth model, while the declining period of coal production (1973–2001) approximates to the exponential-decay model. In a nutshell, the appropriate mathematical models for approximating coal production in Nigeria have been established.

It is expected that the findings from this investigation would contribute in planning for the rebirth of coal as an alternative source of energy.

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Appendix

Enugu coal production (1916–2001)

Year	Production, metric tonnes	Year	Production, metric tonnes	Year	Production, metric tonnes
1916	24,511	1946–47	633,852	1976–77	249,446
1917	83,405	1947–48	551,706	1977–78	246,192
1918	145,407	1948–49	610,283	1978–79	188,806
1919	137,844	1949–50	526,613	1979–80	153,005
1920	180,122	1950–51	583,487	1980–81	114,875
1921–22	194,073	1951–52	566,393	1981–82	63,122
1922–23	112,818	1952–53	613,374	1983	52,730
1923–24	175,137	1953–54	679,437	1984	83,461
1924–25	220,161	1954–55	675,918	1985	139,744
1925–26	242,582	1955–56	750,058	1986	151,214
1926–27	353,274	1956–57	790,030	1987	110,161
1927–28	245,303	1957–58	846,526	1988	113,516
1928–29	363,743	1958–59	905,397	1989	80,973
1929–30	347,115	1959–60	684,800	1990	77,502
1930–31	327,681	1960–61	565,681	1991	51,238
1931–32	263,540	1961–62	596,502	1992	78,912
1932–33	259860	1962–63	615,681	1993	27,686
1933–34	234,296	1963–64	600,229	1994	13,153
1934–35	258,893	1964–65	698,502	1995	19,505
1935–36	257,289	1965–66	730,183	1996	15,310
1936–37	310,308	1966–67		1997	20,766
1937–38	391,159	1967–68	Civil war	1998	18,473
1938–39	323,266	1968–69	Civil war	1999	n.a.
1939–40	300,000	1969–70	Civil war	2000	2,712
1940–41	318,594	1970–71	24,400	2001	2,712
1941–42	402,640	1971–72	264,258	2002	1,771
1942–43	463,978	1972–73	323,001	2003	n.a.
1943–44	528,421	1973–74	314,457	2004	n.a.
1944–45	668,158	1974–75	250,769		
1945–46	505,568	1975–76	257,832		

Sources: This coal production data is compiled from the following sources:
Ugwu (1996) for the 1916–1987 coal production data.
Federal Office of Statistics (FOS) (1991) for the 1988–1990 coal production data.
Federal Office of Statistics (FOS) (1996) for the 1991–1993 coal production data.
Federal Office of Statistics (FOS) (2001) for the 1994–1998 coal production data.
Solid Mineral Production. (2006) for 1999 to 2003 coal production data.

Nomenclature

C	Coal output at the beginning year (for linear models).	Metric tonnes
e	Natural logarithm operator with value = .7183	—
k	Growth constant/coefficient	—
$-k$	Decay constant/coefficient	—
m	Positive gradient (growth) for linear model	—
$-m$	Negative gradient (decay) for linear model	—
P	Coal output at any given year (linear model)	Metric tonnes
P_o	Coal output at the beginning year (for exponential model)	Metric tonnes
P_t	Coal output at any given year (for exponential model)	Metric tonnes
t	Time	Years
x	Value on x-axis	—
y	Value on y-axis	—