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EMPLOYING INFERENTIAL STATISTICAL METHODS FOR DETERMINING RELIABILITY AND APPLICABILITY OF EXPERIMENTAL DATA: AN EMPERICAL ANALYSIS

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ABSTRACT

Tests of hypotheses using the t-test criterion were undertaken to determine the reliability of the data from three thermometers, electronic, alcohol-in-glass, and mercury-in-glass employed for temperature measurements in a pipe heat loss experiment. The t-test results showed that statistically there was no significant difference between the data from electronic and mercury-in-glass thermometers; but there was a significant difference between the temperature from electronic and alcohol-in-glass thermometers. Therefore, the temperatures from the alcohol-in-glass thermometer are rejected, while the temperatures captured by the electronic thermometer were used for heat losses computations. Another test of hypothesis was carried out using the F-test criterion which determined the equality of the means of the temperatures from tank exit, tank entry and room environment. The F-test showed that the means of these three temperatures were unequal. This suggested that their applicability to the evaluations of heat losses would not produce invalid/zero quantity of heat.

Keywords: inferential-statistical methods, electronic thermometer, mercury-in-glass thermometer, alcohol-in-glass thermometer, hypotheses tests, empirical analysis.

INTRODUCTION

All over the world, the results of measurements from empirical experiments are influenced by the limitations inherent in the instruments used. Examples, if an experimenter uses different instruments from different manufacturers, it may be noticed that the instruments indicate varying readings for the same parameter of interest. The varying readings are outcomes of the limitations inherent in the instruments used. The limitations inherent in measuring instruments are sources of instrument error.

Assuming that no error originate from the experimenter then all the errors from empirical measurements will be instrument-based. However, instrument-based errors are beyond the control of the experimenter. Interestingly, there are two known types of instruments' error; systematic error and random error. Systematic error may originate from incorrect instrument design or manufacture, or from inappropriate instrument calibration, or from use of unsuitable method of measurement. On the other hand, the second type of instrument error emanate from the difficulty of exact repetition of the measuring procedure. Even if the acquire skill and regular practice of the experimenter, or the use of computer for empirical data capture, can minimize random error, it may not be completely eliminated.

In view of these, considerations for reliability and applicability of experimental data can be drawn from inferential statistics. Peters and Timmerhaus (1981)[1] explains the application of statistical tests for the treatment of experimental data. Some of the statistical tests are t-test, chi-square test and F-test, all of which involve decision making. And decision making when reliability is the crucial factor follows a scientific process and model. This paper will exploit:

- The t-test in making the decision for reliability of experimental data
- The F-test in making the decision for applicability of experimental data.

To this end, the empirical case study centres on the use of three types of thermometers; electronic, mercury-in-glass, and alcohol-in-glass thermometers, for the experimental investigation of heat transfer modes, in a pipe bearing hot circulating water, drawn from a heated sump tank by a current-driven centrifugal pump. From statistical methods, the reliability of the thermometer measurements is determined by the t-statistic, while the precision of the thermometer readings is measured by the standard deviation. For six different degrees of valve openings the tank exit and inlet temperatures are recorded using the electronic, alcohol-in-glass and mercury-in-glass thermometers, respectively. The six observations(n) obtained from each of the three thermometers at varying degrees of valve openings are conspicuously different from each other. Consequently, the choice of the thermometer readings will therefore be based <u>not</u> on personal feelings but on the results of the inferential statistical analysis involving hypotheses tests for reliability and applicability of the various thermometer measurements.

The aim of the research hypotheses is to make a decision as to whether to uphold or reject the equality of the means of the readings obtained from the three different thermometers. In this regard, two different hypotheses are formulated. The first hypothesis is:

$$H_o: \mu_a = \mu_b$$

$$H_1: \quad \mu_a \neq \mu_b$$

where the symbols are defined in the Notation Section.

The t-test criterion which provides the method for comparing two means will be applied.

The second hypothesis is:

$$H_o: \mu_{b1} = \mu_{w1} = \mu_x$$

$$H_1: \quad \mu_{b1} \neq \mu_{w1} \neq \mu_x$$

where the symbols are defined in the Notation Section.

The F-test model which permits the comparison of two or means by the technique of variance analysis will be employed

Hypothesis tests are decision making tests which follow a scientific process and model in determining the acceptance or rejection of measured data. The result of the test may be true in which case the hypothesis is accepted as valid, or false in which case the hypothesis is rejected as being invalid. The statistical tools which will be employed for the test of hypothesis are the t-test and F-test criteria.

The t-test equation for a two-sample test for means (where n< 30) takes the form, Peters and Timmerhaus (1981)[1] and Nwabuokei (1986)[2]

$$t = \frac{(x_1 - x_1) - 0}{\sqrt{\frac{S_{p1}^2}{n_1} + \frac{S_{p2}^2}{n_2}}} = \frac{x_1 - x_2}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \quad ...(1)$$

EXPERIMENTAL

Hypotheses tests follow a well defined scientific procedure and model.

A two-tailed test of the difference between the two means from the sets of data obtained from electronic and alcohol-in-glass thermometers on one hand, and electronic and mercury-in-glass thermometers on the other hand, will be carried out using t-distribution. The step-by-step procedure presented by Nwabuokei (1986)[2] was adopted as follows:

Step1: *The null and alternative hypotheses were H_0: $\mu_a = \mu_b$; meaning there was no significant difference in the averages of the measurements recorded with electronic and alcohol-in-glass thermometers on one hand, and electronic and mercury-in-glass thermometers on the other hand (i.e equality of means). H_1: $\mu_a \neq \mu_b$; meaning that there was a significant difference in the averages of the measurements recorded with electronic and alcohol-in-glass thermometers on one hand, and electronic and mercury-in-glass thermometers on the other hand (i.e inequality of means).*

Steps 2: *The level of significance was stated $\alpha = 0.05$.*

Steps 3: *The number of degrees of freedom was determined from the formula d.f. = $n_1 + n_2 - 2$.*

The critical values, $t_{\alpha/2}$ for degrees of freedom which corresponded to the level of significance was obtained from the table.

Step 4: *The sample statistic X_1 and X_2; S_1 and S_2; S_p are computed using equations (3), (4) and (2).*

Step5: *The test statistic was computed by applying equation(1)*

Step6: *The value of the test statistic t, obtained in step 5 was compared with the critical value read from table in step 3.*

Step7: *Decision was taken on the criterion; reject H_o if $t > t_{\alpha/2}$, or if $t < -t_{\alpha/2}$, otherwise accept H_o*

Another statistical analysis undertaken involved testing of the difference among the temperatures, T_{bl}, T_{wl} and T_∞ drawn from the electronic thermometer recordings. The F- test was applied because it provided the procedure for testing the equality of three or more means by a statistical technique known as the analysis of variance (ANOVA). Nwabuokei (1986)[2] summarized the steps for one-way analysis of variance for the case of equal sample sizes as follows.

Step 1: *The null and alternative hypotheses were stated.*

H_0: $\mu_{bl} = \mu_l = \mu$

H_1: $\mu_{bl} \neq \mu_l \neq \mu$

Step 2: *The significance level was chosen to be $\alpha = 0.05$*

Step 3: *The F-value with (k -1) and k(r-1) degrees of freedom was read off from the F-table in Appendix C. This was the critical value of the test.*

Step 4: *Random samples of equal size ($n_1 = n_2 = n_3 = r$) were drawn from each of k normal populations.*

Step 5: *SST and SSB were computed; and SSW was determined from SSW = SST - SSB.*

Step 6: *The ANOVA table was constructed and the F- ratio = MSB/MSW was found.*

All ... defined in the Notation Section.

The F-test criterion is known as Analysis of variances (ANOVA). Nwabuokei (1981)[2] defines ANOVA as a statistical technique of into useful components which provide means of measuring different sources of variation.

Step 7: *The F-ratio and the F-value were compared and a decision taken. The null hypothesis was rejected if F-ratio ≥ F-value (k-1), k(r-1), otherwise, it was accepted.*

RESULTS AND DISCUSSION

The temperature measurements obtained from the experiments were analysed for both the t-test and F-test by following the step by step procedures outlined in the previous section.

The two-tailed test of the difference between two temperature means obtained/recorded from electronic and alcohol-in-glass thermometers, using the t-distribution was performed as follows. The average temperatures recorded by the electronic and alcohol-in-glass thermometers were represented by μ_1 and μ_2 respectively. The performance of the test and the decision obtained were outlined below.

(i) $H_o: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

(ii) Level of significance, $\alpha = 0.05$

(iii) Since $n_1 = 6$, and $n_2 = 6$, the number of degrees of freedom was d. f. $= n_1 + n_2 - 2 = 6 + 6 - 2 = 10$. From the t-distribution table, the critical values, $t_{\alpha/2}$ for 10 degrees of freedom were $t_{0.05/2} = \pm 2.228$

(iv) From Appendices I and II, the sample means were, $\bar{X}_1 = 72.75$ and $\bar{X}_2 = 67.22$; and the sample standard deviations were, $S_1^2 = 1.065$ and $S_2^2 = 0.6014$; and the pooled estimated standard deviation was, $S_p^2 = 0.7488$.

(v) The test statistic was eqn(1) for t-distribution

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - 0}{\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}}$$

Substituting in to the equation

$$t = \frac{(72.75 - 67.22) - 0}{\sqrt{\frac{0.7488}{6} + \frac{0.7488}{6}}}$$

$$t = \frac{5.53}{\sqrt{0.1248 + 0.1248}} = \frac{5.53}{0.50}$$

$\therefore$ $t = 11.06$

(vi) The value of the test statistic computed was compared with the critical value obtained from the table, ie $t = 11.06 > t_{0.05/2} = \pm 2.228$

vii) **Decision:** H_o was rejected and H_1 was accepted, and it was concluded that there was a significant difference between the average temperatures recorded by the electronic and alcohol-in-glass thermometers.

Another two-tailed test of the difference between two average temperature (μ_1) obtained from the electronic thermometer, and the average temperature, (μ_3) recorded from the mercury-in-glass thermometer using the t-distribution was performed as follows.

(i) $H_o: \mu_1 = \mu_3$

$H_1: \mu_1 \neq \mu_3$

(ii) $\alpha = 0.05$

(iii) The critical values, $t_{\alpha/2}$ for 10 degrees of freedom was obtained from the t-distribution at Mason et al(1999)[3], $t_{0.05/2} = \pm 2.228$

iv) From Appendix I, the sample means were, $\bar{X}_1 = 72.75$ and $\bar{X}_3 = 72.42$. From Appendix III, the sample standard deviations were $S_1^2 = 1.1358$ and $S_3^2 = 0.4847$; and the pooled estimate of standard deviation was, $S_p^2 = 0.8103$

Table 1: One-way Analysis of Variance

Source of Variation	Sum of squares(ss)	Degrees of Freedom (df)	Mean Squares(ms)	F-ratio
Between groups	SSB = 6678.13	$df_1 = 2$	MSB = 3339.07	F-ratio = $\frac{MSB}{MSW}$ = $\frac{3339.07}{0.732}$ = 4561.57
Within groups	SSW=10.98	$df_2 = 15$	MSW = 0.732	
Total	**SST=6689.11**	**17**	-	-

(vii) The F-ratio and the F-value were compared.

F-ratio = 4561.57 > F-value = 3.68

Decision: The null hypothesis that $\mu_{bt} = \mu_{t} = \mu_{v}$ was rejected. It was concluded that there are significant differences among the average temperature records of T_{b1}, T_{w1} and T_{v}.

CONCLUSION

Inferential Statistics provides the tools and methods for decision making that is void of the experimenter's feeling and bias. This study exploited the tools and methods of Inferential Statistics in making a choice among the readings obtained from electronic, alcohol-in-glass and mercury-in-glass thermometers.

The decision from the t-test for electronic versus alcohol-in-glass thermometers is that there exists a significant different between the mean temperatures recorded by these thermometers. On the other hand, the decision from the t-test for electronic versus mercury-in-glass thermometers is that the means of the temperatures measured by these thermometers are equal. Based on these reliability results, it is concluded that the alcohol-in-glass thermometer is unreliable and therefore unsuitable for the empirical investigation carried out.

After the decision from the F-test result proves that the significant differences in the temperatures of T_{b1}, T_{w1} and T_{v} are not chance occurrence. In this regard, these temperatures are suitable and therefore applicable for the empirical study executed.

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[2] Nwabuokei, Precious O. (1986). Fundamentals of Statistics, New edition With Answers, Koruna Books, Enugu Nigeria, pp.263, 102, 283, 284, 280.

[3] Mason, Robert D., Lind, Douglas A., and Marchal, William G. (1999). Statistical Techniques in Business and Economics, Tenth edition, Irwin McGraw-Hill, Boston, p. 779.

NOTATIONS

T_w	Wall temperature at a particular location	°C
T_v	Temperature of environment (surroundings)	°C
T_b	Bulk temperature at a particular location	°C
T_1	Temperature of surface 1	°C

T_2	Temperature of surface 2	°C
X_1	Sample mean of a set of experimental data	-
X_2	Sample mean of another set of experimental data	-
n_1	Number of measurements in a set of experimental data (sample size)	-

APPENDICES

DATA COMPUTATIONS FOR STATISTICAL ANALYSIS

Appendix I: Computations of Sample Statistic $\overline{X}_1$, $\overline{X}_2$ and $\overline{X}_3$ for Electronic, Alcohol-in-Glass and Mercury-in-Glass Thermometers.

Degree of valve opening	T_{b1}: records from electronic thermometer, (X_1)	T_{b2}: records from alcohol-in-glass thermometer, (X_2)	T_{b3}: records from mercury-in-glass thermometer, (X_3)
90	73.3	68.0	73.0
75	73.0	66.5	73.0
60	73.8	67.8	73.2
45	70.7	68.0	71.3
30	72.1	66.0	71.9
15	73.6	67.0	72.1
Σx_i	436.5	403.3	434.5
$\overline{X}_i = \frac{\Sigma x_i}{N}$	72.75	67.22	72.42
	ie $\overline{X}_1$ = 72.75	$\overline{X}_2$ = 67.22	$\overline{X}_3$ = 72.42

Eqns (3) and (4) are applied.

Eqn (3): $X = \frac{\Sigma x_i}{n}$; where $n_1 = 6$ and $n_2 = 6$

Eqn (4): $S = \sqrt{\frac{\Sigma(x_i - \overline{x})^2}{n}}$

Appendix II: Computations of Sample Statistic, S_1 and S_2; and S_p^2 For Electronic and Alcohol-in-Glass Thermometers

(X_1)	$(X_1-\overline{X}_1)$	$(X_1-\overline{x}_1)^2$	X_2	$(X_2-\overline{X}_2)$	$(X_2-\overline{X}_2)^2$
73.3	0.55	0.3025	68.0	0.78	0.6084
73.0	0.25	0.025	66.5	-0.72	0.5184
73.8	1.05	1.1025	67.8	0.58	0.3364
70.7	-2.05	4.2025	68.0	0.78	0.6084
72.1	-0.65	0.4225	66.0	-1.22	1.4884
73.6	0.85	0.7225	67.0	-0.22	0.0484
	$\Sigma(x_1 - \overline{x}_1)^2$	6.8150		$\Sigma(x_2 - \overline{x}_2)^2$	3.6084
$S_1 = \sqrt{\frac{6.8150}{6}} = 1.0658$			$S_2 = \sqrt{\frac{3.6084}{6}} = 0.6014$		

Eqn (2): $S_p^2 = \frac{(n_1 - 1)(S_1^2) + (n_2 - 1)(S_2^2)}{n_1 + n_2 - 2}$

Substituting

$Sp^2 = \frac{(6-1)(1.0658)^2 + (6-1)(0.6014)^2}{6 \times 6 - 2}$

$= \frac{(5)(1.1359) + (5)(0.3617)}{10}$

$= \frac{5.6795 \times 1.8085}{10}$

$\Rightarrow S_p^2 = 0.7488$

Appendix III: Computations of Sample Statistic, S_1 and S_3; and S_p^2 for Electronic and Mercury-in-Glass Thermometers.

(X_1)	$(X_1-\bar{X}_1)$	$(X_1-\bar{x}_1)^2$	X_3	$(X_3-\bar{X}_3)$	$(X_3-\bar{X}_3)^2$
73.3	0.55	0.3025	73.0	0.58	0.3364
73.0	0.25	0.0625	73.0	0.58	0.3364
73.8	1.05	1.1025	73.2	0.78	0.6084
70.7	-2.05	4.2025	71.3	-1.12	1.2544
72.1	-0.65	0.4225	71.9	-0.52	0.2704
73.6	0.85	0.7225	72.1	-0.32	0.1024
	$\Sigma(x_1-\bar{x}_1)^2$	6.8150		$\Sigma(x_3-\bar{x}_3)^2$	2.9084
$S_1^2 = \frac{\Sigma(x_1-\bar{x}_1)^2}{n}$			$S_3^2 = \frac{\Sigma(x_3-\bar{x}_3)^2}{n}$		
$S_1^2 = \frac{6.8150}{6} = 1.1358$			$S_1^2 = \frac{2.9084}{6} = 0.4847$		

Eqn (2): $S_p^2 = \frac{(n_1 - 1)(S_1^2) + (n3 - 1)(S_3^2)}{n_1 \times n_2 - 2}$

$= \frac{(6-1)(1.1358) + (6-1)(0.4847)}{6 \times 6 - 2}$

$= \frac{(5)(1.1358) + (5)(0.4847)}{10}$

$= \frac{5.679 + 2.4235}{10}$

$S_p^2 = 0.8103$

Appendix IV: Computations of Sample Statistic, $\overline{X}_1$, $\overline{X}_2$, $\overline{X}_3$ and $\overline{\overline{X}}$.

N (i)	T_{b1} (x_{i1})	T_{b2} (x_{i2})	T_{∞} (x_{i3})
1	75.0	73.3	33.0
2	74.8	73.0	33.2
3	75.4	73.8	33.2
4	73.1	70.7	33.0
5	75.1	72.1	32.9
6	74.8	73.6	32.1
Total Sample mean	$\Sigma x_{i1} = 448.2$ $\overline{x}_1 = \frac{\Sigma x_{i1}}{n_1} = 74.70$	$\Sigma x_{i2} = 436.5$ $\overline{x}_2 = \frac{\Sigma x_{i2}}{n_2} = 72.75$	$\Sigma x_{i3} = 197.4$ $\overline{x}_3 = \frac{\Sigma x_{i3}}{n_3} = 32.9$
	$\overline{\overline{X}} = \frac{74.70 + 72.75 + 32.9}{3} = 60.12$		

Appendix V: Computations of SST, SSB, SSW, MSB, MSW and F-ratio
Section 1: Computation of SST

Eqn (8): $SST = \sum_j \sum_i x_{ij}^2 - \frac{T^2}{rk}$

where X_{ij} = all the observations in Table 7-6 starting from X_{11} to x_{63}

$T = \Sigma x_{i1} + \Sigma x_{i2} + \Sigma x_{i3}$ = overall total observations

$= 448.2 + 436.5 + 197.4 = 1082.1$

$r = 6$; and $k = 3$

$SST = (75.0)^2 + (74.8)^2 + (75.4)^2 + (73.1)^2 + (75.1)^2 + (74.8)^2$
$+ (73.3)^2 + (73.0)^2 + (73.8)^2 + (70.7)^2 + (73.6)^2 + (72.1)^2$
$+ (33.0)^2 + (33.2)^2 + (33.2)^2 + (33.0)^2 + (32.9)2 + (32.1)^2$
$- \frac{(1082.1)^2}{6x3}$

SST = 5625 + 5595.04 + 5685.16 + 5343.61 + 5640.01 + 5595.04
\+ 5372.89 + 5329 + 5446.44 + 4998.49 + 5198.41 + 5416.96
\+ 1089 + 1102.24 + 1102.24 + 1089 + 1082.41 + 1030.41
\- 65052.25

SST = 71,741.38 – 65,052.25 = 6689.10

Section 2: Computation of SSB.

Eqn (9): $SSB = \frac{\sum T_j^2}{r} - \frac{T^2}{rk}$

Substituting into eqn (9)

$$SSB = \frac{(448.2)^2 + (436.5)^2 + (197.4)^2}{6} - \frac{(1082.1)^2}{6x3}$$

$$SSB = \frac{200{,}883.24 + 190{,}532.25 + 38{,}966.76}{6} - 65{,}052.25$$

$SSB = 6678.13$

Section 3: Computation of SSW

$SSW = SST - SSB$: Eqn (10)

Substituting into eqn (10)

$SSW = 6689.10 - 6678.13$

$SSW = 10.98.$

Section 4: Computation of Mean Squares and F-ratio

Eqn (6): $MSB = \frac{SSB}{k-1}$

Substituting into eqn (6),

$MSB = \frac{6678.13}{3-1} = 3339.07$

Eqn (7): $\frac{SSW}{k(r-1)} = MSW$

Substituting into eqn (7),

$MSW = \frac{10.98}{3(6-1)} = 0.732$

Eqn (5): $F\text{-ratio} = \frac{MSB}{MSW}$

Substituting into eqn (5),

$F\text{-ratio} = \frac{3339.07}{0.732} = 4561.57$